

Definition 0.1. Let us recall that as a [Quantum Algebraic Topology object](#), a [quantum automaton](#) is defined by the *quantum triple* $Q_A = (\mathcal{G}, \mathcal{H} - \mathfrak{R}_G, \text{Aut}(\mathcal{G}))$, where $\mathcal{G}$ is a (locally compact) [quantum groupoid](#), $\mathcal{H} - \mathfrak{R}_G$ are the unitary [representations](#) of $\mathcal{G}$ on [rigged Hilbert spaces](#) $\mathfrak{R}_G$ of quantum states and [quantum operators](#) on the [Hilbert space](#) $\mathcal{H}$, and $\text{Aut}(\mathcal{G})$ is the transformation, or *automorphism groupoid* of quantum transitions that represents all flip-flop quantum transitions of one qubit each between the permitted quantum states of the quantum automaton.

With the data from above definition we can now define also the category of quantum automata as follows.

Definition 0.2. The *category of quantum automata* $\mathcal{Q}_A$ is defined as an [algebraic category](#) whose objects are triples $(\mathcal{H}, \Delta : \mathcal{H} \rightarrow \mathcal{H}, \mu)$ (where $\mathcal{H}$ is either a Hilbert space or a rigged Hilbert space of quantum states and [operators](#) acting on $\mathcal{H}$, and μ is a measure related to the [quantum logic](#), LM , and (quantum) transition probabilities of this quantum [system](#)), and whose [morphisms](#) are defined between such triples by [homomorphisms](#) of Hilbert spaces, $\Omega : \mathcal{H} \rightarrow \mathcal{H}$, naturally compatible with the operators Δ , and by homomorphisms between the associated [Haar measure](#) systems.

An alternative definition is also possible based on *Quantum Algebraic Topology*.

Definition 0.3. A *quantum algebraic topology* definition of the [category](#) of quantum [algebraic automata](#) involves the objects specified above in **Definition 0.1** as quantum automaton triples (Q_A) , and [quantum automata](#) homomorphisms defined between such triples; these Q_A morphisms are defined by [groupoid homomorphisms](#) $h : \mathcal{G} \rightarrow \mathcal{G}^*$ and $\alpha : \text{Aut}(\mathcal{G}) \rightarrow \text{Aut}(\mathcal{G}^*)$, together with unitarity preserving mappings u between unitary representations of $\mathcal{G}$ on rigged Hilbert spaces (or [Hilbert space bundles](#)).